Modal Analysis of the Thermal Transfer between a Building and the Surrounding Ground

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ABSTRACT

In this paper we present a detailed procedure for the thermal analysis of a building including thermal transfer through the surrounding ground.

In order to solve this problem two techniques have been used: decomposition of the building and the surrounding ground into subsystems (a unidimensional sub-system for each wall and a bidimensional subsystem for the ground), and modal analysis and reduction of each subsystem.

The matrices characterizing the reduced subsystems are stored in a library. The thermal behavior of the building can be evaluated using the following steps: construction of a global thermal system using agregated subsystems, and iterative resolution of the equations.

The principal advantages of this method are: efficient computer time, non-linear problems can be treated, and matrices of aggregated subsystems can be computed once for all and then stored in a library.

INTRODUCTION

Thermal bridges problems, especially those due to the connections between the building and the surrounding ground, are frequently

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encountered. They are important because a lot of buildings have a large part of their envelope in contact with the ground and now that above-ground thermal insulation has become more efficient, losses through the ground are proportionately more important than before and thus justify more attention.

The heat transfers through ground floor foundations and the ground are three-dimensional. Most of existing code exploitations now use two dimensions, or even steady-state conditions (Fauconnier and Grelat 1984).

Indeed the exploitations of calculation codes dynamically for long periods and with hourly time steps would be lengthy even with elaborate means (it should not be forgotten that we are working systems making use of about 1000 differential equations).

Another approach has been developed (Achard et al. 1983, 1986) using normal coordinates (Gough 1982) and harmonic transfer functions. It leads to analytic solutions that can only be established for simple configurations. A similar approach, by decomposing the governing heat conduction equation into steady-state and periodic components, gives nondimensional results (scaled up for any climate) concerning only periodic foundation heat flow (Shen et al. 1988).

We present a general method based on substructuration of our thermal system (building + ground), on modal analysis, and on reduction of each subsystem. The main advantage of such a methodology is that it limits the number of calculations thus saving a lot of computer time and allowing the use of microcomputers.

SUBSTRUCTURATION

The methodology used is illustrated in Figure 1. It shows that the global thermal system (building + ground) can be divided into different subsystems ("above grade walls" and "floor + ground"). It is assumed that the conduction heat transfer through walls is unidimensional, while through the ground it is unidimensional.

Each matricial subsystem is reduced after applying modal analysis on each (Sicard et al. 1985).
The simulation of the global system is reached by connecting all the subsystems using surface temperatures as interconnection variables.

MODAL ANALYSIS OF THE SUBSYSTEMS, "WALL"

Hypothesis and Basic Definitions

The thermal system is assumed to be linear, stationary, and reciprocal (i.e., thermophysical coefficients are time independent, internal and boundary heat-transfers are linearized, and symmetry of local transfers is verified). If we assume the conduction heat transfer to be unidimensional, we have to solve the following equation for each slab of the wall:

$$a \frac{\delta^2 T}{\delta x^2} = \frac{\delta T}{\delta t} \qquad (1)$$

where
a = diffusivity of the slab
T = temperature (time and space function).

Using a finite-difference method the previous differential equation can be discretized. Then an approximation of Equation 1 becomes, for each node i (cf. Figures 2 and 3 for definition of terms) of the concerned domain:

$$C_i \frac{\delta T_i}{\delta t} = \frac{(T_{i-1} - T_i)}{R_{i-1,i}} + \frac{(T_{i+1} - T_i)}{R_{i,i+1}} \qquad (2)$$

where
C_i = capacity of node number i
T_i = temperature of node number i
$R_{i,i+1}$ = thermal resistivity between mode i and i+1

The energy balance for all the nodes of the network can be summarized in a matricial form:

$$[C]\ (\mathring{T}) = [A]\ (T) + [D]\ (u) \qquad (3)$$

where
(T) = vector of nodes temperature
[C] = diagonal thermal capacity matrix
[A] = symmetric internal heat-transfer matrix
[D] = external heat-transfer matrix
(u) = excitations vector

Two matrices, [W] and [P], can be obtained by the diagonalization of the matrix $[C]^{-1}$ [A]:

$$[W] = [P]^{-1}\ [C]^{-1}\ [A]\ [P]$$

where
[W] = diagonal matrix of eigenvalues (λ k)
[P] = eigenvectors matrix

Then we can define the time constants of the thermal system: $\tau_k = 1/|\lambda_k|$. (In our case, eingenvalues and eingenvectors are negative reals.)

Equation 4 expresses the relation between a certain number of outputs (Y), the state vector (T), and the excitation vector (u):

$$\{Y\} = [J]\ \{T\} + [G]\ \{u\} \tag{4}$$

If we suppose the following relation:

$$\{T\} = [P]\ \{X\}$$

where
$\{Y\}$ = output vector
$[J]$ = system observation matrix
$[G]$ = matrix of static responses
$\{X\}$ = dynamic excitation of the modes

Equations 3 and 4 can be written as:

$$\begin{aligned} \{\overset{\circ}{X}\} &= [W]\ \{X\} + [B]\ \{u\} \\ \{Y\} &= [H]\ \{X\} + [G]\ \{u\} \end{aligned} \tag{5}$$

where
$[B] = [P]^{-1}\ [C]^{-1}\ [D]$

and: $[H] = [J]\ [P]$

If we are interested in the q^{th} outputs of the thermal system due to the p^{th} excitation (assimilated to a step function) then (Bacot 1984):

$$\frac{Y_{qp}(t) - Y_{qp}(0)}{Y_{qp}(\infty) - Y_{qp}(0)} = 1 - \sum_{k=1}^{n} Z_k \exp(-t/\tau k) \tag{6}$$

with $$Z_k = \frac{H_{qk}\ B_{kp}\ \tau_k}{Y_j(\infty) - Y_j(0)}$$

<u>Reduction of the Subsystem "Wall"</u>

Several investigations have been made in order to reduce the number of state equations governing the physical phenomena (Michailesco 1979; Bacot 1984; Sicard 1984). For conduction heat transfer in walls, we adopt a two-stage procedure: first, we retain only those eingenvectors that meet the following criteria:

$$\tau_k \geq \delta t/4 \tag{7}$$

(if δt is the choosen time step, then the terms $\exp(-\delta t/\tau k)$ are negligible).

In the second stage, we keep among the previously selected eigenvectors those that satisfy the condition:

$$|Z_k| \times \exp(-\delta t/\tau_k) \geq \epsilon \qquad (8)$$

where ϵ is a defined precision.

Let us see the matricial formulation of the reduced model. Equation 5 can be written:

$$\begin{Bmatrix} \overset{\circ}{X}_1 \\ \overset{\circ}{X}_2 \end{Bmatrix} = \begin{bmatrix} W_1 & \\ & W_2 \end{bmatrix} \begin{matrix} X_1 \\ X_2 \end{matrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} (u)$$

$$\text{and } (Y) = [H_1 \quad H_2] \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} + [G]\ (u) \qquad (9)$$

where $[W_1]$ = eigenvalues bloc verifying the two previous criteria Equations 7 and 8 and $[W_2]$ the other part of $[W]$.

Then the solution of Equation 5 can be approximated by:

$$\begin{aligned} (X_1) &= [W_1]\ (X_1) + [B_1]\ (u) \\ (\tilde{Y}) &= [H_1]\ (X_1) + [G']\ (u) \end{aligned} \qquad (10)$$

where
$[G'] = [G] - [H_2]\ [W_2]^{-1}\ [B_2]$

and $(\tilde{Y})$ = a good approximation for (Y).

Example of a Two-Component Wall

We present here the application of such a procedure to a two-component wall (concrete + insulation) for which spectra are defined in Figure 4. For an hour time step, δt, the complete solution ($(Y) = \phi(o,t)$) and reduced solution ((Y)) are compared in Figure 5. The two solutions are almost superimposed. We notice that the largest error is for small times ($t < \delta t$). For this example only three differential equations were required (i.e., 13 equations were dropped) and the calculation time was considerably decreased.

MODAL ANALYSIS OF THE SUBSYSTEM "GROUND"

Hypothesis and Basic Definitions

As has already been said, heat transfer through floor, foundations, and soil is a three-dimensional (3D) problem. However, it is sometimes possible to solve the problem with a two-dimensional (2D) approach (Fauconnier and Grelat 1984; Walton

1987) and often a 3D problem can be shown to be a combination of two 2D problems. In the present study, the problem is supposed to be two dimensional.

The boundary conditions are summarized in Figure 6 and an example of the associated network is shown in Figure 7. A finite element method (Allard 1978; Achard et al. 1986) leads us to a matricial formulation analogous to the one written for the "wall" subsystem.

$$[C]\ \{\mathring{T}\} = [A]\ \{T\} + [D]\ \{u\} \qquad (11)$$

[C], the thermal capacity matrix, is not diagonal anymore but band-diagonal.

The output vector has two components (cf. Figure 6 for a definition of terms) :

-- T_{Sc} = the central average "surface temperature"
-- T_{Sp} = the peripheral average "surface temperature"

Then:

$$\{Y\} = \begin{Bmatrix} T_{Sc} \\ T_{Sp} \end{Bmatrix} = [R]\ [T] \qquad (12)$$

[R] = system observation matrix

Reduction of the Subsystem "Ground"

According to Table 1, the time constants of this subsystem vary from a few seconds to several years. We distinguish three scales in the series of constant times τ_i:

-- fast evolution of the system :

$$\tau N < \tau_i < \tau_R => n_3 \text{ eigenvectors}$$

(N is the number of nodes of the network: 399 in the presented examples).

-- slow evolution of the system:

$$\tau_i > \tau_L => n_1 \text{ eigenvectors}$$

-- intermediary evolution of the system:

$$\tau_R < \tau_i < \tau_L => n_2 \text{ eigenvectors}$$

with $n_1 + n_2 + n_3 = N$.

The particular values τ_R and τ_L are chosen in order to satisfy the following two criteria:

$$\mathrm{Exp}\ (-\delta t/\tau_L) = 1 - \epsilon \qquad (13)$$

and

$$\mathrm{Exp}\ (-\ \delta t/\tau_R) = 0 + \epsilon \qquad (14)$$

The calculated values of τ_R and τ_L depend on the desired precision ϵ.

Considering the change of state vector using the relation {T} = [P] {X}, the solution of the problem is:

$$X(t_i) = \exp\ ([W].\delta t).\{X(t_{i-1})\}$$
$$+\ (\exp\ ([W].\delta t) - [I])\ [W]^{-1}\ [B]\ \{u(t_i)\} \qquad (15)$$

where $t_i = i.\delta t$

Using the previously mentioned subdivision, the solution can be approximated by :

-- for the n_1 first eigenvectors:

$$X_1\ (t_i) = \exp\ (w^* t)\ \{X_1(0)\}$$
$$+\ [\exp\ (w^* t) - 1]\ [W_1]^{-1}\ [B_1]\ \{u(t_i)\} \qquad (16)$$

where
w^* = an average eingenvalue characterizing the slow subsystem and satisfying criterion (13);

-- for the n_{ll} last eigenvectors:

$$X_3\ (t_i) = -\ [W_3]^{-1}\ [B_3]\ \{w(t)\} \qquad (17)$$

-- for the n_2 intermediary eigenvectors:

$$X_2(t_i) = \exp\ ([W_2].t)\ \{X_2(0)\}$$
$$+\ (\exp\ ([W_2].t) - [I])\ [W_2]^{-1}\ [B_2]\ \{u(t)\} \qquad (18)$$

SUBSYSTEMS COUPLING

The subsystems are connected by surface temperatures and conduction fluxes (Figure 8). The resolution procedure is:

-- Energy balances for the indoor air and for each indoor surface give the indoor air temperature T_{AI} and the vector (T_S) of indoor surface temperatures. (Conduction fluxes are assumed to be known for each subsystem.)

-- Evaluation of the conduction heat transfer for each subsystem knowing the surface temperatures for each.

-- Iteration of the equations to satisfy a convergence criterion.

Figure 8 summarires this procedure. The iterative procedure is especially interesting because it allows the treatment of non-linear problems (radiation between surfaces, air infiltration ...).

Evaluation of the conduction fluxes through the walls can be done by other equivalent methods in terms of calculation time (response factors, Z transform ...). (Perez Sanchez 1989) but it is not the purpose of this paper to discuss these methods.

VALIDATION AND EXAMPLES

Figure 9 shows very few differences between the results of the complete model (20 nodes for each wall and 399 nodes for "ground") and the reduced model (3 state equations for walls and about 50 for "ground").

The code AMABAT we have presented (Mokhtari 1987; Mokhtari et al. 1987) has been compared with another approach (TONYm) developped in this laboratory (Roldan 1986; Perez Sanchez et al. 1987).

In this second code (TONYm), conduction heat-transfer through walls is evaluated using the Z-transform method and for conduction heat transfer through ground using an analytical approach to define an equivalent wall (Achard et al. 1986; Perez Sanchez et al. 1987). Figure 10 shows that, although methods of calculation are completely different, the results are almost similar. In order to illustrate what problems can be treated with that code, let us look at the results applied to three different configurations (Figure 11).

In figure 12 the comparison between loads of two identical buildings is presented with the only difference being the width of the floor insulation (Figure 11). Globally, we can say that "case 3" loads are about 10 % higher than "case 4" loads. In Figure 13, energy requirements for each week are compared for configurations 4 and 5 (Figure 11). We notice that, if coupling with the ground is an advantage during the winter (Figure 13), it can be a disadvantage during the summer.

CONCLUSIONS

This study shows that it is possible to include dynamic models taking into account conduction heat transfer through the ground in general building code thermal behavior. Complex configurations can be treated and a library of reduced models for standard configurations can be constituted. The methodology we have presented allows the calculation of frequently encountered non-linear problems. The present work should be extended in order to take into account the fact that it is not always possible to treat a three-dimensional problem like a two-dimensional one, and the fact that it would be very useful to be able to consider different zones of the building (a room or group of rooms).

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"GROUND" TIME CONSTANTS					
N°_	YEARS	MONTHS	HOURS	MINUTES	SECONDS
1	07.0	05.0	11.0	09.0	46.0
2	04.0	02.0	02.0	12.0	41.0
3	01.0	10.0	21.0	19.0	51.0
4	00.0	10.0	06.0	14.0	02.0
5	—	09.0	08.0	15.0	14.0
6	—	07.0	13.0	18.0	10.0
7	—	06.0	11.0	06.0	18.0
8	—	04.0	03.0	15.0	10.0
9	—	01.0	16.0	57.0	55.0
10	—	01.0	09.0	21.0	34.0
11	—	01.0	03.0	03.0	31.0
12	—	00.0	18.0	10.0	47.0
:	:	:	:	:	:
42	—	—	01.0	01.0	56.0
43	—	—	00.0	22.0	13.0
:	:	:	:	:	:
102	—	—	—	01.0	05.0
103	—	—	—	00.0	56.0
:	:	:	:	:	:
163	—	—	—	—	15.0
:	:	:	:	:	:
325	—	—	—	—	01.0
:	:	:	:	:	:
399	00.0	00.0	00.0	00.0	00.5

Table 1. Example of "Ground" sub-system time constants

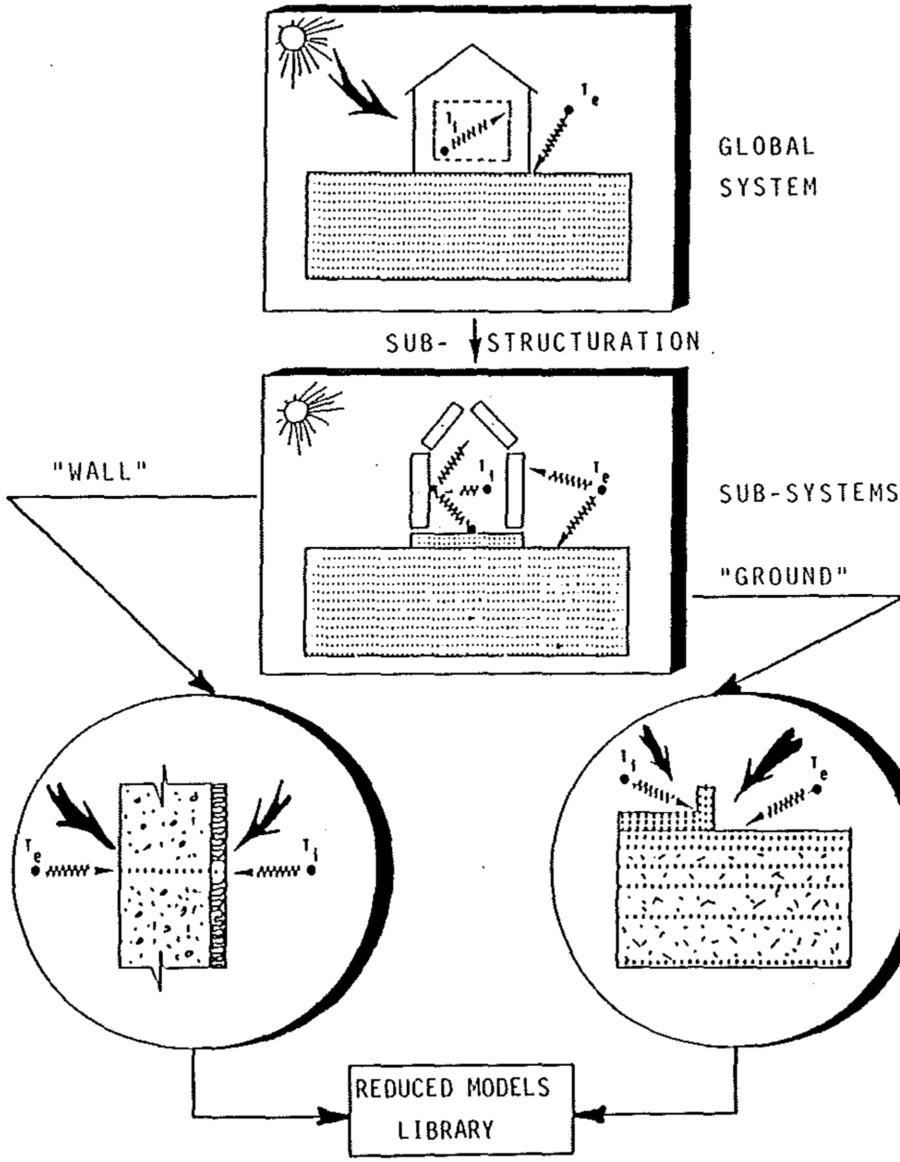

Figure 1. Sub-structuration

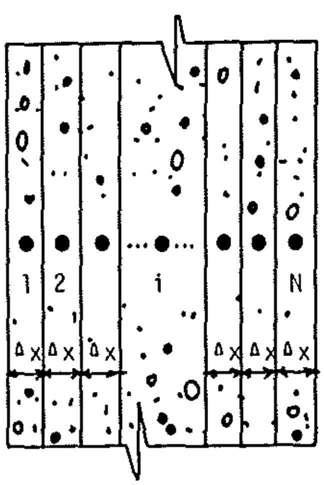

Figure 2. Spatial discretization

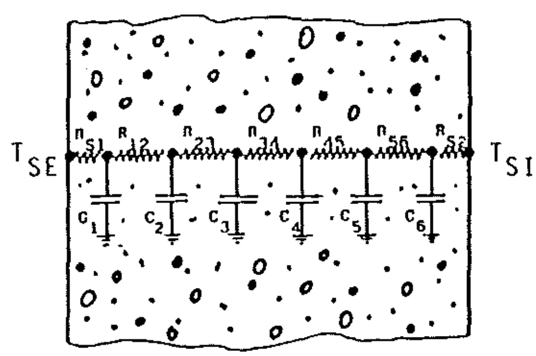

Figure 3. Equivalent electrical network

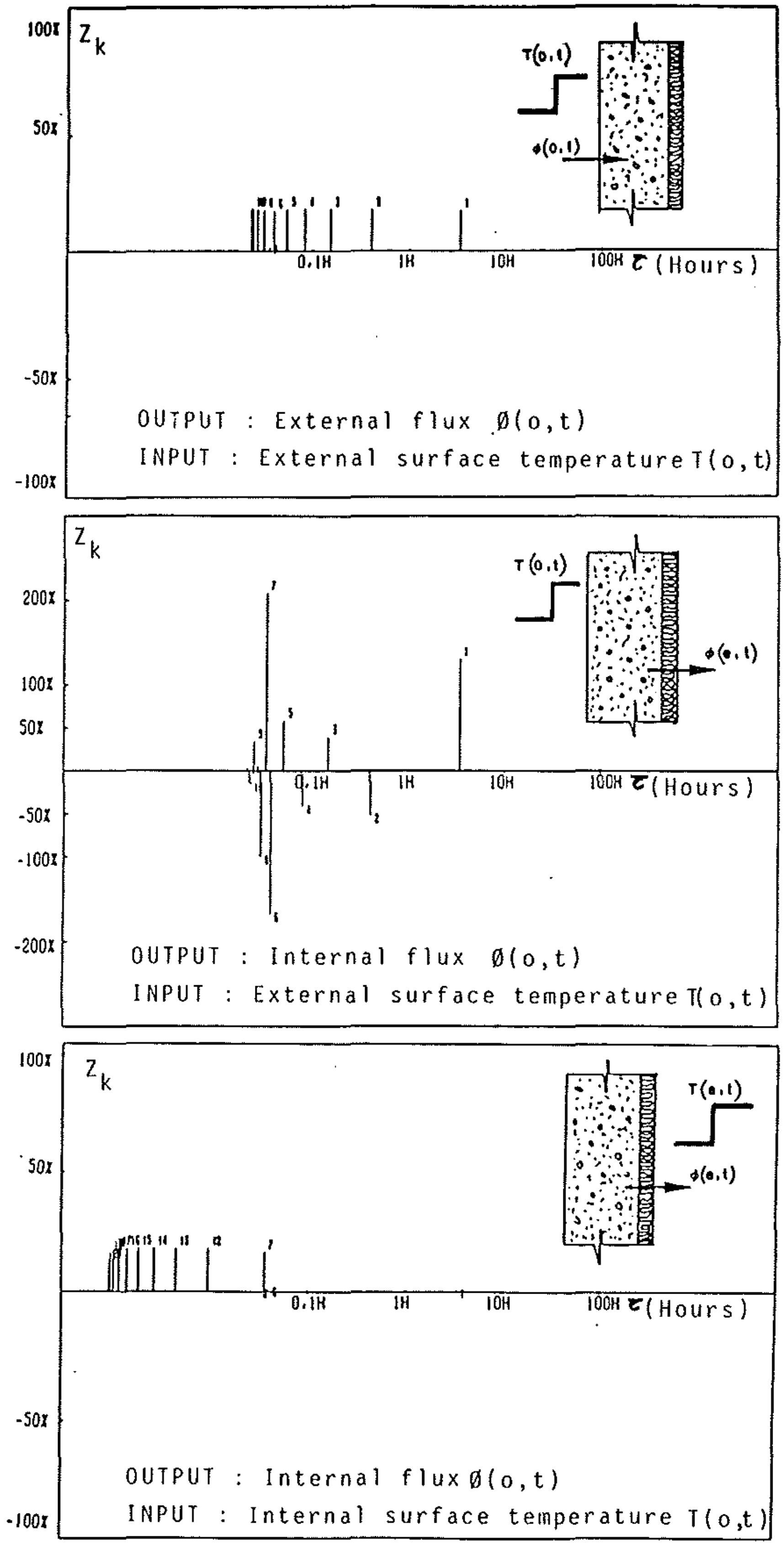

Figure 4. Different INPUT/OUTPUT spectra

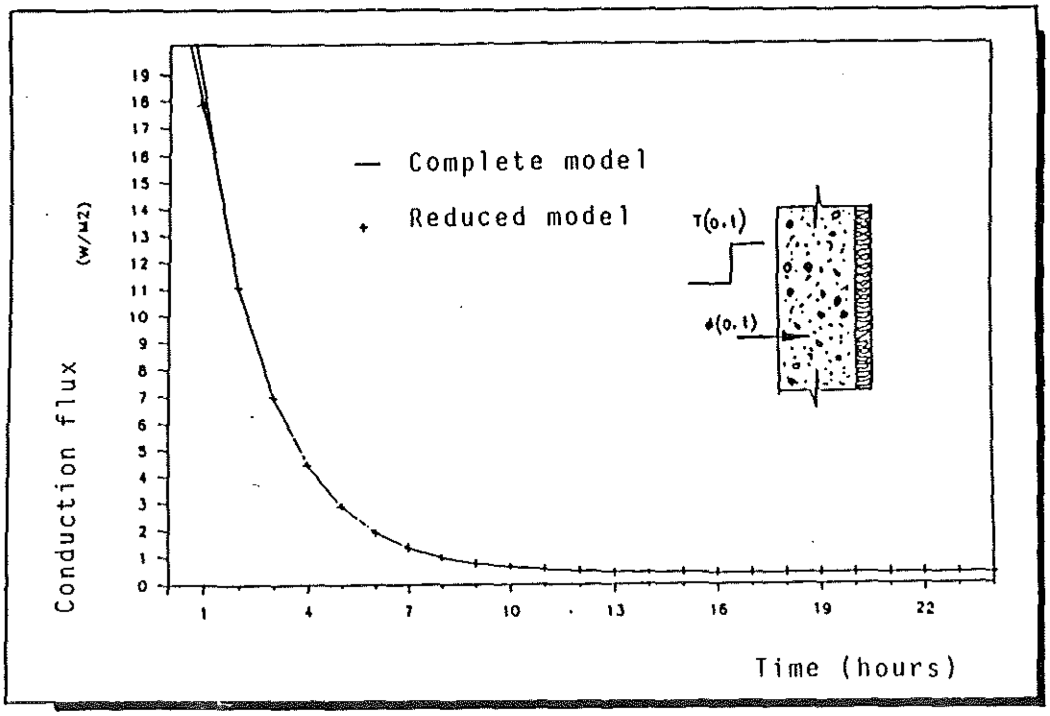

Figure 5. Comparison complete/reduced model. Input: T(o,t)/output: Ø(o,t)

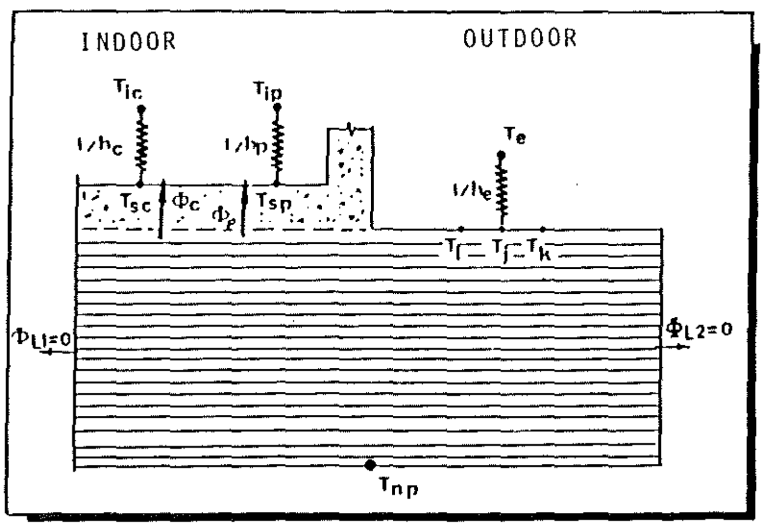

Figure 6. Boundary conditions

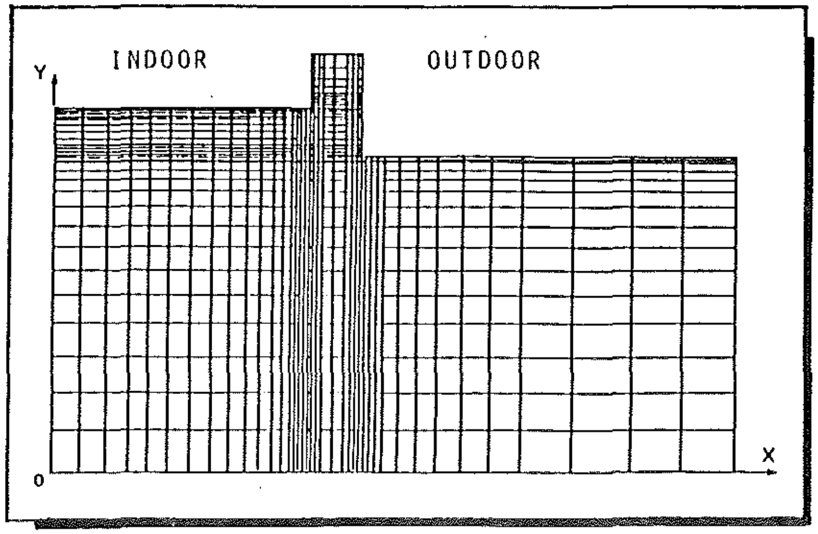

Figure 7. Discretization of the domain (399 nodes)

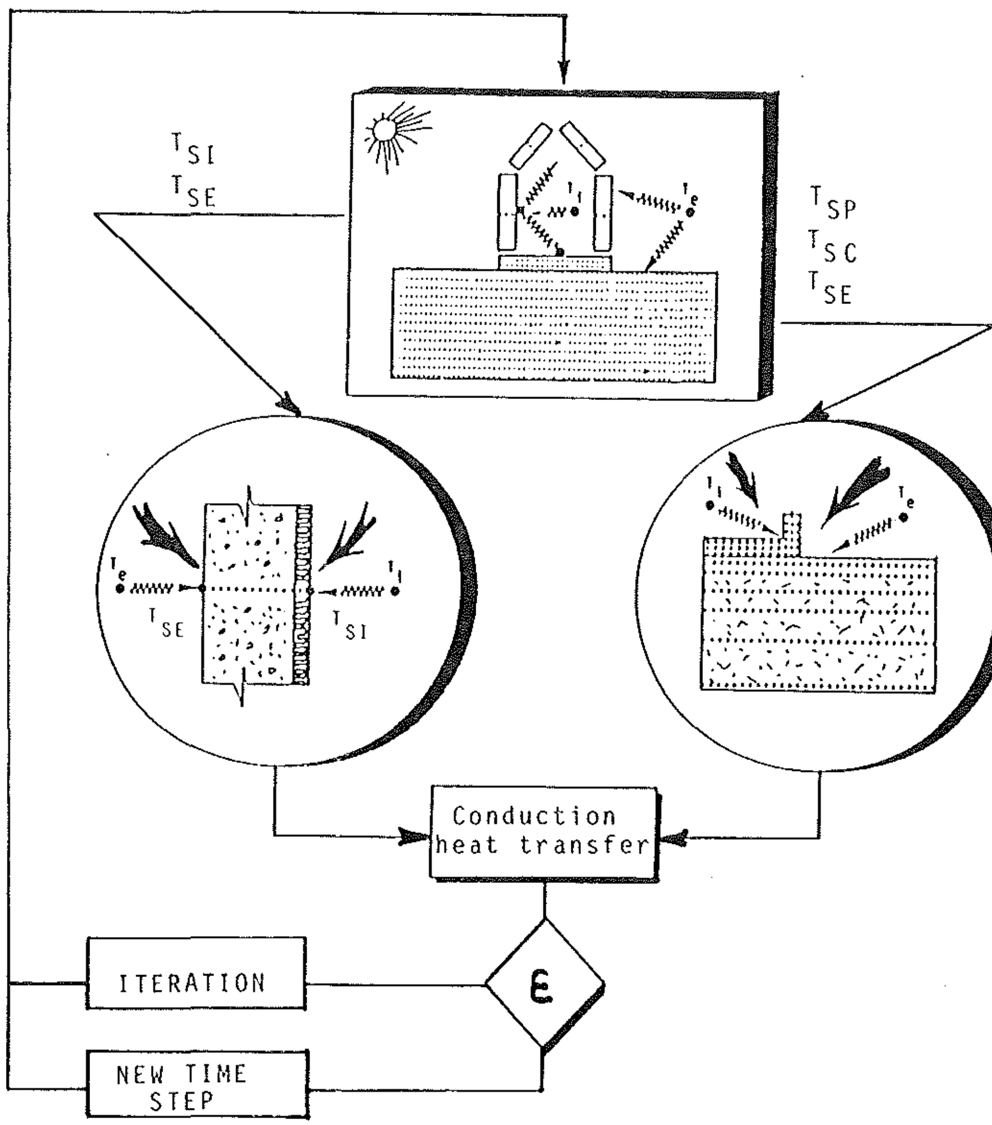

Figure 8. Interconnection procedure

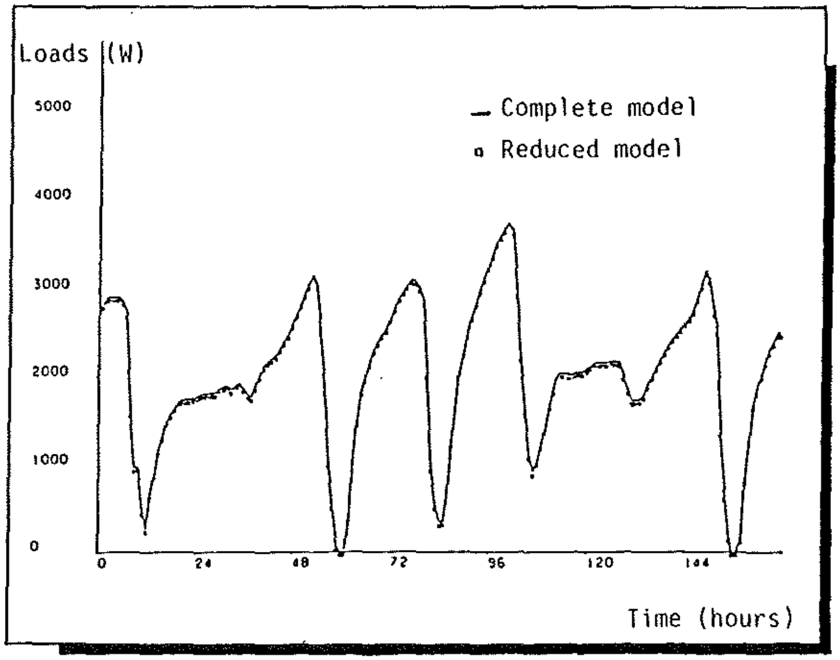

Figure 9. Comparison between complete and reduced model

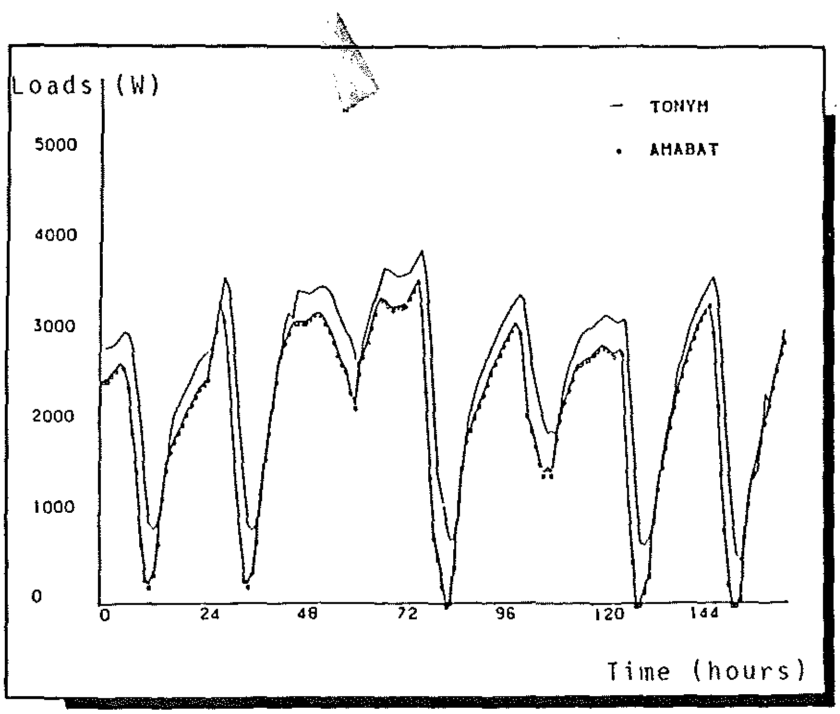

Figure 10. Comparison of two different approaches

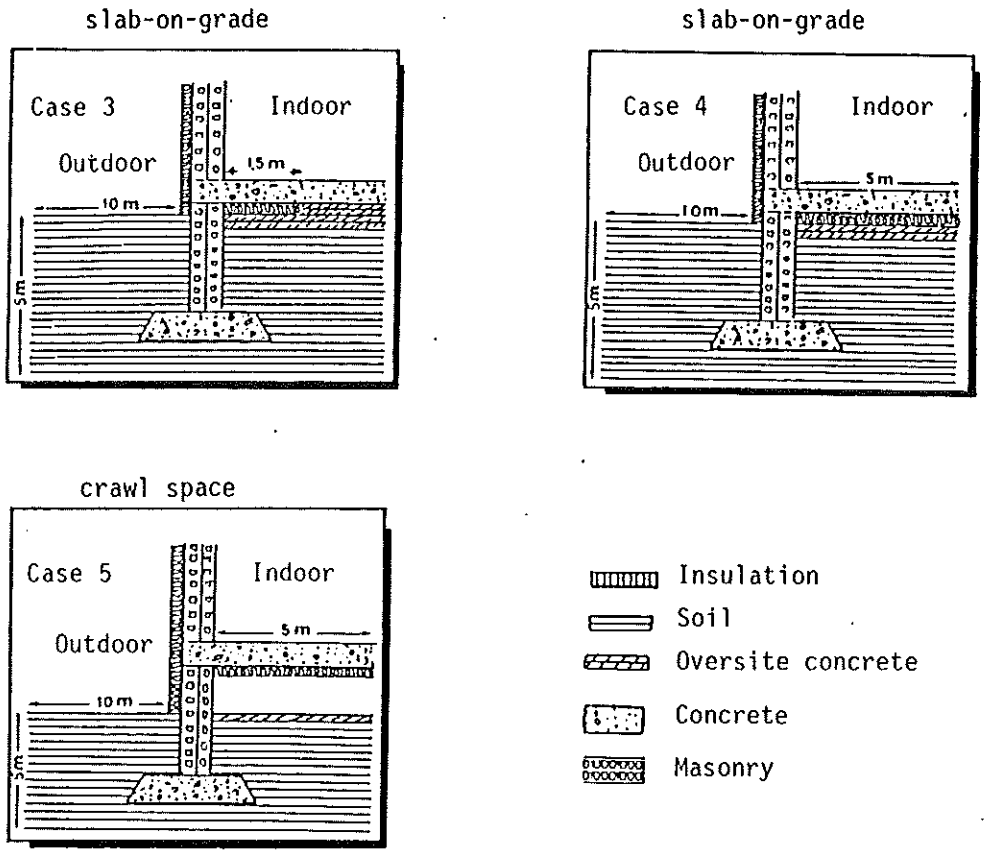

Figure 11. Three example configurations

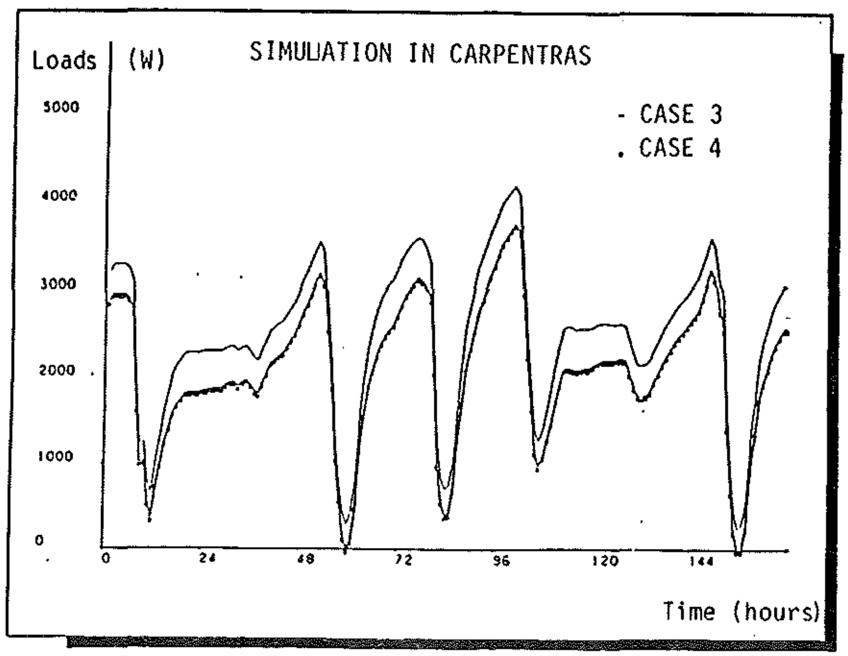

Figure 12. Effect of insulation width

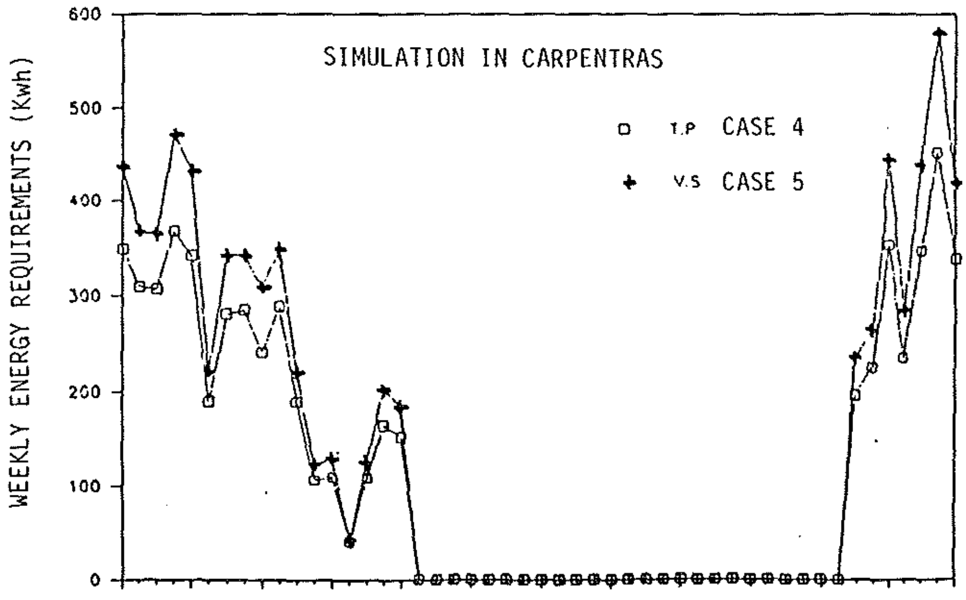

Figure 13. Comparison of Case 4/Case 5 configurations